

Group Theory and Its Applications

Yi Lu

Draft of September 9, 2026

Work in progress. These notes are written up as the course runs; several chapters are still an outline only, and nothing here has been through a careful revision. Corrections are welcome.

ACKNOWLEDGEMENT. These notes draw in part on a $\LaTeX$ transcription that Kong Shuo (孔硕), then a student on this course, made from the 2025 lectures and from my handwritten notes. His careful and unprompted work is used throughout the earlier chapters and is gratefully acknowledged.

Contents

Principal texts	v
I. Groups	1
1. Introduction	3
1.1. Three themes of twentieth-century physics	3
1.2. What symmetry is, and how to measure it	3
1.3. Discrete symmetry	4
1.3.1. Mirror symmetry in a molecule	4
1.3.2. Particle exchange	5
1.3.3. Crystals	5
1.4. Continuous symmetry and Noether's theorem	6
2. Groups: basic definitions and examples	9
2.1. Definition and first examples	9
2.2. Subgroups, order, and cyclic groups	10
2.3. Equivalence relations and residue classes	11
2.4. Direct products and Abelian groups	12
2.5. Matrix groups	12
2.6. Generators and presentations	16
2.7. Application: the Pauli group and stabilizer codes	18
3. Homomorphism and isomorphism	23
3.1. Homomorphism and isomorphism	23
3.2. Kernel and image	24
3.3. Examples	25
3.4. $SU(2)$ and $SO(3)$	26
3.5. $GL(V)$ and $GL(n, \kappa)$	27
4. Group actions	29
4.1. Actions, G -sets, and orbits	29
4.2. Induced action on function spaces	29
4.3. Equivariant maps	29
4.4. Orbit-stabilizer theorem	29
4.5. Conjugacy, centralizers, and normalizers	29
4.6. The class equation	29
4.7. p -groups and Cauchy's theorem	29
5. The symmetric group	31
5.1. Permutations, cycles, and transpositions	31
5.2. Cayley's theorem	31

5.3. Parity and the alternating group	31
5.4. The braid group	31
6. Cosets, Lagrange, and conjugacy	33
6.1. Cosets and Lagrange's theorem	33
6.2. Conjugacy	33
6.3. Class functions and characters	33
6.4. Conjugacy classes of S_n and Young diagrams	33
7. Normal subgroups, quotients, and extensions	35
7.1. Normal subgroups	35
7.2. Quotient groups	35
7.3. The isomorphism theorems	35
7.4. Exact sequences	35
7.5. Central extensions and the finite Heisenberg group	35
II. Representation theory	37
8. Representations	39
8.1. Motivation: why representations	39
8.2. Basic definitions	39
8.3. Examples of representations	39
8.4. Operations on representations	39
9. Unitarity and reducibility	41
9.1. Unitary representations and the Haar measure	41
9.2. The regular representation	41
9.3. Reducible and irreducible representations	41
10. Schur's lemmas and orthogonality	49
10.1. Schur's lemmas	49
10.1.1. Schur's lemmas I and II	49
10.2. Orthogonality relations and the Peter–Weyl theorem	56
10.2.1. Motivation and importance	56
10.2.2. Preliminaries: $L^2(G)$ as a $G \times G$ representation	57
10.2.3. The Peter–Weyl theorem	58
III. Applications	71
11. Point groups and crystal field theory	73
11.1. Crystallographic point groups	73
11.2. Character tables	73
11.3. Splitting of atomic orbitals	73
11.4. Weak and strong field limits	73
11.5. Hybridization and molecular orbitals	73
12. Light–matter interaction and selection rules	75
12.1. Dipole selection rules	75

12.2. Spectroscopy of the cuprates	75
12.3. Symmetry of superconducting order parameters	75

Principal texts

- [Moore] G. W. Moore. *Abstract Group Theory. Physics 618: Applied Group Theory*. Lecture notes, Chapter 1; later chapters dated 2018–2019. 2023.
URL: <https://www.physics.rutgers.edu/~gmoore/618Spring2023/GTLect1-AbstractGroupTheory-2023.pdf> (visited on 09/08/2026).
- [Zee] A. Zee. *Group Theory in a Nutshell for Physicists*. Princeton: Princeton University Press, 2016. elementary; a good entry point.
- [Dres] M. S. Dresselhaus, G. Dresselhaus, and A. Jorio. *Group Theory: Application to the Physics of Condensed Matter*. Berlin: Springer, 2008. many worked physical applications.
- [Tao] 陶瑞宝. 物理学中的群论. 高等教育出版社, 2011. a standard Chinese textbook.
- [Ma] 马中骐. 物理学中的群论. 科学出版社, 2006. a standard Chinese textbook.
- [Chen] 陈金全. 群表示论的新途径. 上海: 上海科学技术出版社, 1984. Nanjing University.