P 9. $T: \mathbb{C}^2 \to \mathbb{C}^2$ equivariant $\Rightarrow T(\vec{z}) = \alpha \vec{z}$. $\alpha \in \mathbb{C}$

$$\begin{tikzcd} \mathbb{C}^2 \arrow[r, "T"] \arrow[d, "SU(2)"'] & \mathbb{C}^2 \arrow[d, "SU(2)"] \\ \mathbb{C}^2 \arrow[r] & \mathbb{C}^2 \end{tikzcd}$$

$M \cdot T(\vec{z}) = T(M \cdot \vec{z})$, $\forall \vec{z} \in \mathbb{C}^2$, $\forall M \in SU(2) \Rightarrow [T, M] = 0$

$T = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ $M_1 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ $M_2 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$

$TM_1 = M_1 T \Rightarrow \begin{pmatrix} b & -a \\ d & -c \end{pmatrix} = \begin{pmatrix} -c & -d \\ a & b \end{pmatrix} \Rightarrow T = \begin{pmatrix} a & b \\ -b & a \end{pmatrix}$

$TM_2 = M_2 T \Rightarrow \begin{pmatrix} bi & ai \\ ai & -bi \end{pmatrix} = \begin{pmatrix} -bi & ai \\ ai & bi \end{pmatrix} \Rightarrow b = 0 \Rightarrow T = a\mathbb{1}_2$

$\Rightarrow T(\vec{z}) = \alpha \vec{z}$, $\alpha \in \mathbb{C}$

P 10. $D_3 \cong S_3$ $D = \langle a, b \mid a^3 = b^2 = (ab)^2 = 1 \rangle$

$\varphi: D_3 \to S_3$

$\varphi(a) = (123)$

$\varphi(b) = (12)$

P 11. (1) $q = \begin{pmatrix} 1 & 2 & 3 & \cdots & n \\ n & n-1 & n-2 & \cdots & 1 \end{pmatrix}$

$$= \begin{cases} (1\,n)(2\;n-1)\cdots\left(\frac{n-1}{2}\;\frac{n+3}{2}\right) & n \text{ odd} \\ (1\,n)(2\;n-1)\cdots\left(\frac{n}{2}\;\frac{n}{2}+1\right) & n \text{ even} \end{cases}$$

$$= \prod_{i=1}^{\lceil \frac{n-1}{2} \rceil} (i,\, n+1-i)$$

(2) $\lceil \frac{n-1}{2} \rceil$ even $\Leftrightarrow$ $n = 4k,\, 4k+1$ $\quad (k \in \mathbb{N})$

odd $\Leftrightarrow$ $n = 4k+2,\, 4k+3$

(3) recall from lecture that

$(ij) = (i, i+1)(i+1 . j)(i, i+1) \qquad (i < j-1)$

$= \sigma_i\, (i+1, j)\, \sigma_i$

$= \sigma_i \sigma_{i+1}\, (i+2\;\; j)\, \sigma_{i+1} \sigma_i$

alternatively.

$\phi = (n-1, n)(n-2, \cdots n) \cdots (1\,2\,3\,4 - n)$

and $(i, i+1, \cdots j+1) = \sigma_i \sigma_{i+1} \cdots \sigma_j$