Problem 27

Let V^+, V^-, and V^2 denote the trivial, sign, and the two-dimensional standard irreps of S_3, respectively. Show that

(a) $V^+ \otimes V^\mu \cong V^\mu$

(b) $V^- \otimes V^- \cong V^+$

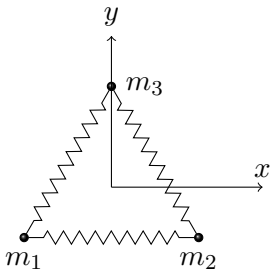
(c) $V^- \otimes V^2 \cong V^2$

(d) $V^2 \otimes V^2 \cong V^+ \oplus V^- \oplus V^2$

Problem 28

Consider the two-dimensional coupled harmonic oscillator depicted below, where the three "atoms" (point masses) can move in plane while keeping their center of total mass fixed. Its symmetry is described by D_3. Look up the character table of point group D_3 online or in textbooks. One example can be found at link here[1].

The vectors of displacements of atoms from their equilibrium positions, $\psi = (\delta x_1, \delta y_1, \delta x_2, \delta y_2, \delta x_3, \delta y_3)^T$, span a representation space for D_3. Determine the character of each conjugacy class under this representation, and find its isotypic decomposition into irreps.



[1] http://symmetry.jacobs-university.de/cgi-bin/group.cgi?group=303&option=4