

P25

$$\text{show } \int_{\mathcal{G}} \chi_{\mu}(\mathcal{g}) \chi_{\nu}(\mathcal{g}^{-1}h) d\mathcal{g} = \frac{\delta_{\mu\nu}}{n_{\mu}} \chi_{\nu}(h)$$

$$\begin{aligned} \text{LHS} &= \int_{\mathcal{G}} \sum_i \mu_{i,i}^{\mu}(\mathcal{g}) \sum_j \left[\sum_k \mu_{j,k}^{\nu}(\mathcal{g}^{-1}) \mu_{k,j}^{\nu}(h) \right] d\mathcal{g} \\ &= \sum_{i,j,k} \mu_{k,j}^{\nu}(h) \int_{\mathcal{G}} \mu_{i,i}^{\mu}(\mathcal{g}) \overline{\mu_{k,j}^{\nu}}(\mathcal{g}) d\mathcal{g} \\ &= \sum_{i,j,k} \mu_{k,j}^{\nu}(h) \frac{1}{n_{\mu}} \delta_{\mu\nu} \delta_{i,k} \delta_{i,j} \\ &= \frac{\delta_{\mu\nu}}{n_{\mu}} \sum_i \mu_{i,i}^{\nu}(h) = \frac{\delta_{\mu\nu}}{n_{\mu}} \chi_{\nu}(h) \end{aligned}$$

P26

see Moore's lecture notes.