## Problem 27

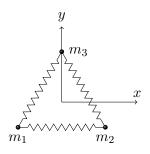
Let  $V^+$ ,  $V^-$ , and  $V^2$  denote the trivial, sign, and the two-dimensional standard irreps of  $S_3$ , respectively. Show that

- (a)  $V^+ \otimes V^\mu \cong V^\mu$
- (b)  $V^- \otimes V^- \cong V^+$
- (c)  $V^- \otimes V^2 \cong V^2$
- (d)  $V^2 \otimes V^2 \cong V^+ \oplus V^- \oplus V^2$

## Problem 28

Consider the two-dimensional coupled harmonic oscillator depicted below, where the three "atoms" (point masses) can move in plane while keeping their center of total mass fixed. Its symmetry is described by  $D_3$ . Look up the character table of point group  $D_3$  online or in textbooks. One example can be found at link here<sup>1</sup>.

The vectors of displacements of atoms from their equilibrium positions,  $\psi = (\delta x_1, \delta y_1, \delta x_2, \delta y_2, \delta x_3, \delta y_3)^T$ , span a representation space for  $D_3$ . Determine the character of each conjugacy class under this representation, and find its isotypic decomposition into irreps.



<sup>&</sup>lt;sup>1</sup>http://symmetry.jacobs-university.de/cgi-bin/group.cgi?group=303&option=4