

**Problem 16**<sup>1</sup>

Let  $D \subset SU(2)$  be the subgroup of diagonal matrices. Note that  $D \simeq U(1)$ .

- (a) Compute explicitly its normalizer  $N_{SU(2)}(D)$ .
- (b) Compute the quotient group  $N_{SU(2)}(D)/D$ .
- (c) Show that conjugation by elements in  $N_{SU(2)}(D)$  acts on elements of  $D$  by a permutation of the diagonal elements, and the permutation only depends on the projection to the quotient.
- (d) Show that there is no subgroup of  $N_{SU(2)}(D)$  whose conjugation on  $D$  induces the permutation action.

**Problem 17**<sup>2</sup>

A group action of  $G$  on  $X$  can be viewed as a homomorphism  $\phi: G \rightarrow S_X$

- (a) Show that the action is effective if and only if the homomorphism is injective.
- (b) Show that the subset of group elements that act ineffectively is a normal subgroup  $H \triangleleft G$ .
- (c) Show that there is an effective action of the quotient group  $G/H$  on  $X$ .

**Problem 18**<sup>3</sup>

Suppose  $G$  is finite and acts transitively on a finite set  $X$  with more than one point. Show that there is an element  $g \in G$  with no fixed points on  $X$ .

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<sup>1</sup>p. 91 of [GM]

<sup>2</sup>p. 133 of [GM]

<sup>3</sup>p. 138 of [GM]