Problem 12 (The complex conjugate representation ¹)

Consider two N-dimensional representations ϕ_1 , ϕ_2 of SU(N) and U(N), where $\phi_1(u) = u$, and $\phi_2(u) = u^*$, show that ϕ_1 , ϕ_2 are

- (1) equivalent for SU(2),
- (2) inequivalent for U(2),
- (3) inequivalent for SU(N) (N > 2).

Problem 13

- (1) Let H be a subgroup of G and [G:H] = 2. Show that $H \triangleleft G$.
- (2) Let Z(G) be the center of group G. Show that if G/Z(G) is cyclic, then G is abelian.

Problem 14

Show that

- (1) $SL(n,\kappa) \triangleleft GL(n,\kappa)$, and
- (2) $A_n \triangleleft S_n$.

Problem 15

For $g_1, g_2 \in G$, define the group commutator

$$[g_1, g_2] = g_1 g_2 g_1^{-1} g_2^{-1}.$$

The commutator subgroup [G,G] is the subgroup generated by all the commutators. Show that

- (1) $[G,G] \lhd G$, and
- (2) If $H \triangleleft G$, then G/H is abelian if and only if [G, G] is a subgroup of H.

¹p. 79 of [GM]