Problem 07

Construct a nontrivial homomorphism from the quaternion group to the Klein four group,

$$\phi : Q \to \mathbb{Z}_2 \times \mathbb{Z}_2.$$

Show its kernel $\ker \phi$ and image $\mathrm{im}\phi$.

Problem 08

Show that the following diagram commutes if and only if $k_1 = k_2 \mod N$.

$$
\begin{array}{ccc}
\mathbb{Z}_N & \xrightarrow{\ m_{k_1}\ } & \mathbb{Z}_N \\
\downarrow{\psi} & & \downarrow{\psi} \\
\mu_N & \xrightarrow{\ p_{k_2}\ } & \mu_N
\end{array}
$$