

P26. show  $\int_{\mathcal{G}} \chi_{\mu}(\mathfrak{g}) \chi_{\nu}(\mathfrak{g}^{-1}h) d\mathfrak{g} = \frac{\delta_{\mu\nu}}{n_{\mu}} \chi_{\nu}(h)$

$$\begin{aligned}
 \text{LHS} &= \int_{\mathcal{G}} \sum_i \mu_{ii}^{\mu}(\mathfrak{g}) \sum_j \left[ \sum_k \mu_{jk}^{\nu}(\mathfrak{g}^{-1}) \mu_{kj}^{\nu}(h) \right] d\mathfrak{g} \\
 &= \sum_{ijk} \mu_{kj}^{\nu}(h) \int_{\mathcal{G}} \mu_{ii}^{\mu}(\mathfrak{g}) \overline{\mu_{kj}^{\nu}}(\mathfrak{g}) d\mathfrak{g} \\
 &= \sum_{ijk} \mu_{kj}^{\nu}(h) \frac{1}{n_{\mu}} \delta_{\mu\nu} \delta_{ik} \delta_{ij} \\
 &= \frac{\delta_{\mu\nu}}{n_{\mu}} \sum_i \mu_{ii}^{\nu}(h) = \frac{\delta_{\mu\nu}}{n_{\mu}} \chi_{\nu}(h)
 \end{aligned}$$

P27 see Moore's lecture notes.