Problem 16[1]

Let $D \subset SU(2)$ be the subgroup of diagonal matrices. Note that $D \simeq U(1)$.

(a) Compute explicitly its normalizer $N_{SU(2)}(D)$.

(b) Compute the quotient group $N_{SU(2)}(D)/D$.

(c) Show that conjugation by elements in $N_{SU(2)}(D)$ acts on elements of D by a permutation of the diagonal elements, and the permutation only depends on the projection to the quotient.

(d) Show that there is no subgroup of $N_{SU(2)}(D)$ whose conjugation on D induces the permutation action.

Problem 17[2]

A group action of G on X can be viewed as a homomorphism $\phi\colon G \to S_X$

(a) Show that the action is effective if and only if the homomorphism is injective.

(b) Show that the subset of group elements that act ineffectively is a normal subgroup $H \lhd G$.

(c) Show that there is an effective action of the quotient group G/H on X.

Problem 18[3]

Suppose G is finite and acts transitively on a finite set X with more than one point. Show that there is an element $g \in G$ with no fixed points on X.

Problem 19

Prove the following Lemma used in the proof of Cauchy's theorem in the lecture:

Let G be a finite *Abelian* group whose order can be divided by a prime number p. Then there exists an element $g \in G$ whose order is p.

[1] p. 91 of [GM]
[2] p. 133 of [GM]
[3] p. 138 of [GM]