## Problem 07

Construct a nontrivial homomorphism from the quaternion group to the Klein four group,

$$
\phi: Q \rightarrow \mathbb{Z}_{2} \times \mathbb{Z}_{2}
$$

Show its kernel $\operatorname{ker} \phi$ and image $\operatorname{im} \phi$.

## Problem 08

Show that the following diagram commutes if and only if $k_{1}=k_{2} \bmod N$.


## Problem 9

Consider the linear action of $S U(2)$ on $\mathbb{C}^{2}$. Show that any linear equivariant map $T$ : $\mathbb{C}^{2} \rightarrow \mathbb{C}^{2}$ is of the form $T(\vec{z})=\alpha \vec{z}$ for some $\alpha \in \mathbb{C}$.

## Problem 10

What is the smallest symmetric group $S_{n}$ that the dihedral group $D_{3}$ can be embedded? Construct the embedding and conclude that $D_{3} \cong S_{3}$.

## Problem 11

A permutation $\phi$ reverses the order of $\{1,2, \ldots, n\}$ to $\{n, n-1, \ldots, 1\}$.
(1) Write down its cycle decomposition.
(2) Is it an even or odd permutation?
(3) Generate it using the generators $\sigma_{i}=(i i+1)$, where $1 \leq i<n$.

