Problem 07

Construct a nontrivial homomorphism from the quaternion group to the Klein four group,

$$\phi: Q \to \mathbb{Z}_2 \times \mathbb{Z}_2.$$

Show its kernel ker ϕ and image im ϕ .

Problem 08

Show that the following diagram commutes if and only if $k_1 = k_2 \mod N$.

$$\mathbb{Z}_N \xrightarrow{m_{k_1}} \mathbb{Z}_N$$

$$\downarrow \psi \qquad \qquad \downarrow \psi$$

$$\mu_N \xrightarrow{p_{k_2}} \mu_N$$

Problem 9

Consider the linear action of SU(2) on \mathbb{C}^2 . Show that any linear equivariant map $T : \mathbb{C}^2 \to \mathbb{C}^2$ is of the form $T(\vec{z}) = \alpha \vec{z}$ for some $\alpha \in \mathbb{C}$.

Problem 10

What is the smallest symmetric group S_n that the dihedral group D_3 can be embedded? Construct the embedding and conclude that $D_3 \cong S_3$.

Problem 11

A permutation ϕ reverses the order of $\{1, 2, \dots, n\}$ to $\{n, n-1, \dots, 1\}$.

- (1) Write down its cycle decomposition.
- (2) Is it an even or odd permutation?
- (3) Generate it using the generators $\sigma_i = (i \ i + 1)$, where $1 \le i < n$.