

Problem 25

$$\begin{aligned}
 (1) \quad T(\mathcal{F})^{\otimes n} [\sigma(\omega_1 \otimes \dots \otimes \omega_n)] &= T(\mathcal{F})^{\otimes n} [\omega_{\sigma_1} \otimes \omega_{\sigma_2} \otimes \dots \otimes \omega_{\sigma_n}] \\
 &= [T(\mathcal{F}) \omega_{\sigma_1}] \otimes [T(\mathcal{F}) \omega_{\sigma_2}] \otimes \dots \\
 &= \sum_{\{j_i\}} \prod_i \pi_i [M(\mathcal{F})]_{\sigma(i)j_i} \omega_{j_1} \otimes \omega_{j_2} \otimes \dots \otimes \omega_{j_n}
 \end{aligned}$$

$$\begin{aligned}
 \sigma [T(\mathcal{F})^{\otimes n} (\omega_1 \otimes \dots \otimes \omega_n)] &= \sigma [T(\mathcal{F}) \omega_1 \otimes \dots \otimes T(\mathcal{F}) \omega_n] \\
 &= \sum_{j_1} M_{1j_1} \omega_{\sigma(j_1)} \otimes \dots \otimes \sum_{j_n} M_{nj_n} \omega_{\sigma(j_n)} \\
 &= \sum_{j_i} \prod_i [M(\mathcal{F})]_{ij_i} \omega_{\sigma(j_1)} \otimes \dots \otimes \omega_{\sigma(j_n)} \\
 &= \sum_{j_i} \prod_i [M(\mathcal{F})]_{i\sigma^{-1}(j_i)} \omega_{j_1} \otimes \dots \otimes \omega_{j_n} \\
 &= \sum_{j_i} \prod_i [M(\mathcal{F})]_{\sigma(i)j_i} \omega_{j_1} \otimes \dots \otimes \omega_{j_n}
 \end{aligned}$$

(2)

$$(13)(12) = (123)$$

1	3
2	

$$c = (e + (13))(e - (12)) = e + (13) - (12) - (123)$$

$$\begin{aligned}
 cV^{\otimes 3} &= \text{span} \{ v_i \otimes v_j \otimes v_k + \sigma_k \otimes v_j \otimes v_i \\
 &\quad - v_j \otimes v_i \otimes v_k - v_j \otimes \sigma_k \otimes v_i \}
 \end{aligned}$$

$$(a_2)_{ijk} = a_{ijk} + a_{kji} - a_{jik} - a_{kij}$$

$$\begin{aligned}
 \Rightarrow \begin{cases} (a_2)_{ijk} + (a_2)_{jki} + (a_2)_{kij} = 0 \\ (a_2)_{ijk} = - (a_2)_{jki} \end{cases}
 \end{aligned}$$

Problem 26

for $SU(3)$, at most 3 rows.

$$\begin{array}{|c|c|} \hline & \\ \hline & \\ \hline \end{array} \otimes \begin{array}{|c|} \hline 1 \\ \hline 2 \\ \hline \end{array} = \begin{array}{|c|c|c|} \hline & & 1 \\ \hline & 2 & \\ \hline \end{array} \oplus \begin{array}{|c|c|c|} \hline & & 1 \\ \hline & & \\ \hline 2 & & \\ \hline \end{array} \oplus \begin{array}{|c|c|} \hline & \\ \hline & 1 \\ \hline 2 & \\ \hline \end{array}$$

not allowed examples:

$$\begin{array}{|c|c|c|c|} \hline & & 2 & 1 \\ \hline & & & \\ \hline & & & \\ \hline \end{array} : \text{not semistandard} \quad \begin{array}{|c|c|} \hline & \\ \hline & 2 \\ \hline 1 & \\ \hline \end{array} : \text{violates (3)}$$

$$\text{dims} : 8 \times 3 = 15 + 6 + 3$$