Problem 25

(1) $T(g)^{\otimes n}[\sigma(w_1\otimes\cdots\otimes w_n)] = T(g)^{\otimes n}(w_{\sigma 1}\otimes w_{\sigma(2)}\otimes\cdots\otimes w_{\sigma(n)})$

$= [T(g)w_{\sigma(1)}]\otimes[T(g)w_{\sigma(2)}]\otimes\cdots$

$= \sum_{\{j_i\}}\prod_i [M(g)]_{\sigma(i)j_i}\, w_{j_1}\otimes w_{j_2}\otimes\cdots\otimes w_{j_n}$

$\sigma[T(g)^{\otimes n}(w_1\otimes\cdots\otimes w_n)] = \sigma[T(g)w_1\otimes\cdots\otimes T(g)w_n]$

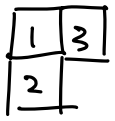
$= \sum_{j_1} M_{1j_1} w_{\sigma(j_1)}\otimes\cdots\otimes\sum_{j_n} M_{nj_n} w_{\sigma(j_n)}$

$= \sum_{j_i}\prod_i [M(g)]_{ij_i}\, w_{\sigma(j_1)}\otimes\cdots\otimes w_{\sigma(j_n)}$

$= \sum_{j_i}\prod_i [M(g)]_{i\sigma^{-1}(j_i)}\, w_{j_1}\otimes\cdots\otimes w_{j_n}$

$= \sum_{j_i}\prod_i [M(g)]_{\sigma(i)j_i}\, w_{j_1}\otimes\cdots\otimes w_{j_n}$

(2) $(13)(12) = (123)$



$c_{(2,1)} = (e+(13))(e-(12)) = e+(13)-(12)-(123)$

$cV^{\otimes 3} = \mathrm{span}\{v_i\otimes v_j\otimes v_k + v_k\otimes v_j\otimes v_i$

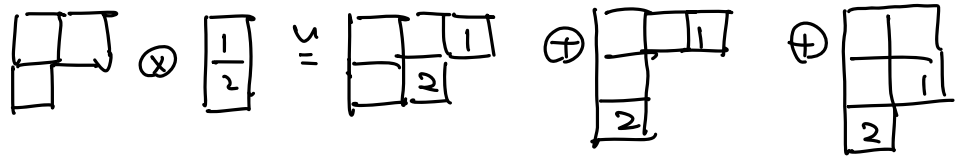
$- v_j\otimes v_i\otimes v_k - v_j\otimes v_k\otimes v_i\}$

$(a_2)_{ijk} = a_{ijk} + a_{kji} - a_{jik} - a_{kij}$

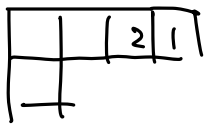
$\Rightarrow \begin{cases} (a_2)_{ijk} + (a_2)_{jki} + (a_2)_{kij} = 0 \\ (a_2)_{ijk} = -(a_2)_{jik} \end{cases}$

Problem 26

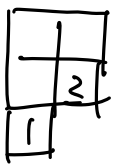
for SU(3). at most 3 rows.



not allowed examples:



: not semistandard



: violates (3)

dims: $8 \times 3 \overset{!}{=} 15 + 6 + 3$