

P22

$$\text{show } \int_{\mathcal{G}} \chi_{\mu}(g) \chi_{\nu}(g^{-1}h) dg = \frac{\delta_{\mu\nu}}{n_{\mu}} \chi_{\nu}(h)$$

$$\begin{aligned} \text{LHS} &= \int_{\mathcal{G}} \sum_{ijk} \mu_{ii}^{\mu}(g) \mu_{jk}^{\nu}(g^{-1}) \mu_{kj}^{\nu}(h) dg \\ &= \sum_{ijk} \mu_{kj}^{\nu}(h) \int_{\mathcal{G}} \mu_{ii}^{\mu}(g) \overline{\mu_{kj}^{\nu}}(g) dg \\ &= \sum_{ijk} \mu_{kj}^{\nu}(h) \frac{1}{n_{\mu}} \delta_{\mu\nu} \delta_{ik} \delta_{ij} \\ &= \frac{\delta_{\mu\nu}}{n_{\mu}} \sum_i \mu_{ii}^{\nu}(h) = \frac{\delta_{\mu\nu}}{n_{\mu}} \chi_{\nu}(h) \end{aligned}$$

P23. Straight-forward computation.