

Problem 21 (Fourier transform)

The Fourier transform of a locally compact commutative group G and its inverse are defined as

$$\begin{aligned}\hat{f}(\chi) &= \int_G f(g)\chi(g)dg \\ f(g) &= \int_{\hat{G}} \hat{f}(\chi)\overline{\chi(g)}d\chi,\end{aligned}$$

where $f \in L^1(G)$, $\hat{f} \in L^1(\hat{G})$.

- (1) Identify \hat{G} for $G = \mathbb{R}, U(1), \mathbb{Z}$, and \mathbb{Z}_N .
- (2) Write down their respective Fourier transforms using normalized Haar measures. Conclude that Fourier transforms from G to \hat{G} and from \hat{G} to $\hat{\hat{G}} = G$ are inverses of each other.