P07. $T: \mathbb{C}^2 \to \mathbb{C}^2$ equivariant $\Rightarrow T(\vec{z}) = \alpha \vec{z}$. $\alpha \in \mathbb{C}$

$$\begin{tikzcd} \mathbb{C}^2 \arrow[r, "T"] \arrow[d, "SU(2)"'] & \mathbb{C}^2 \arrow[d, "SU(2)"] \\ \mathbb{C}^2 \arrow[r] & \mathbb{C}^2 \end{tikzcd}$$

$M \cdot T(\vec{z}) = T(M \cdot \vec{z})$ $\quad [T \cdot M] = 0$

$T = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$ $\quad M_1 = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ $\quad M_2 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}$

$TM_1 = M_1 T \Rightarrow \begin{pmatrix} b & -a \\ d & -c \end{pmatrix} = \begin{pmatrix} -c & -d \\ a & b \end{pmatrix} \Rightarrow T = \begin{pmatrix} a & b \\ -b & a \end{pmatrix}$

$TM_2 = M_2 T \Rightarrow \begin{pmatrix} bi & ai \\ ai & -bi \end{pmatrix} = \begin{pmatrix} -bi & ai \\ ai & bi \end{pmatrix} \Rightarrow b = 0 \Rightarrow T = a \mathbb{1}_2$

$\Rightarrow T(\vec{z}) = \alpha \vec{z}$, $\alpha \in \mathbb{C}$

P08 $D_3 \cong S_3$ $\quad D = \langle a . b \mid a^2 = b^3 = (ab)^2 = 1 \rangle$

$\varphi: D_3 \to S_3$

$\varphi(a) = (12)$

$\varphi(b) = (123)$

P09. (1) $\phi = \begin{pmatrix} 1 & 2 & 3 & \cdots & n \\ n & n-1 & n-2 & \cdots & 1 \end{pmatrix}$

$$= \begin{cases} (1\ n)(2\ n-1)\cdots(\frac{n-1}{2}\ \ \frac{n+3}{2}) & n \text{ odd} \\ (1\ n)(2\ n-1)\cdots(\frac{n}{2}\ \ \frac{n}{2}+1) & n \text{ odd} \end{cases}$$

$$= \prod_{i=1}^{\lceil \frac{n-1}{2} \rceil} (i,\ n+1-i)$$

(2) $\lceil \frac{n-1}{2} \rceil$ even $\Leftrightarrow$ $n = 4k,\ 4k+1$ $(k \in \mathbb{N})$

odd $\Leftrightarrow$ $n = 4k+2,\ 4k+3$

(3) recall from lecture that

$$(ij) = (i, i+1)(i+1 . j)(i, i+1) \qquad (i < j-1)$$

$$= \sigma_i (i+1, j) \sigma_i$$

$$= \sigma_i \sigma_{i+1} (i+2\ j) \sigma_{i+1} \sigma_i$$

alternatively.

$$\phi = (n-1, n)(n-2, \cdots n) \cdots (1\ 2\ 3\ 4 - n)$$

and $(i, i+1, \cdots j+1) = \sigma_i \sigma_{i+1} \cdots \sigma_j$