

1.  $\forall a, b, ab \in G. \quad a = a^{-1}. \quad b = b^{-1}$

$$\Rightarrow (ab)^2 = (ab)(ab) = (ab)(a^{-1}b^{-1}) = e$$

$$\Rightarrow ab = (a^{-1}b^{-1})^{-1} = ba$$

2.  $(H \text{ is a subgroup}) \stackrel{\text{iff}}{\Leftrightarrow} (e \in H \ \& \ h_1, h_2 \in H \Rightarrow h_1 h_2^{-1} \in H)$

$\Rightarrow$  trivial by def. of (sub)group.

$$\Leftarrow e, h \in H. \quad \Rightarrow \quad e \cdot h^{-1} = h^{-1} \in H \quad (\text{exists inverse})$$

$$h_1 \cdot h_2 \in H. \quad \Rightarrow \quad h_2^{-1} \in H. \quad \Rightarrow \quad h_1 (h_2^{-1})^{-1} = h_1 h_2 \in H.$$

(closure)

3. see P15 of Greg Moore notes [GM].