Problem 10 (The complex conjugate representation [1])

Consider two N-dimensional representations ϕ_1, ϕ_2 of $SU(N)$ and $U(N)$, where $\phi_1(u) = u$, and $\phi_2(u) = u^*$, show that ϕ_1, ϕ_2 are

(1) equivalent for $SU(2)$,

(2) inequivalent for $U(2)$,

(3) inequivalent for $SU(N)$ $(N > 2)$.

Problem 11 (From the lecture)

(1) Let H be a subgroup of G and $[G : H] = 2$. Show that $H \lhd G$.

(2) Let $Z(G)$ be the center of group G. Show that if $G/Z(G)$ is cyclic, then G is abelian.

Problem 12 (A few normal subgroups)

Show that

(1) $SL(n, \kappa) \lhd GL(n, \kappa)$, and

(2) $A_n \lhd S_n$.

Problem 13 (Commutator subgroup)

For $g_1, g_2 \in G$, define the *group commutator*

$$[g_1, g_2] = g_1 g_2 g_1^{-1} g_2^{-1}.$$

The commutator subgroup $[G, G]$ is the subgroup *generated* by all the commutators. Show that

(1) $[G, G] \lhd G$, and

(2) If $H \lhd G$, then G/H is abelian if and only if $[G, G]$ is a subgroup of H.

[1] p. 60 of [GM]