Problem 04 (pp. 19-20 of [GM] [1])

Let $\mathbf{q} = \{q_1, q_2, \ldots, q_n\}^T$ and $\mathbf{p} = \{p_1, p_2, \ldots, p_n\}^T$ be coordinates and momenta for a classical mechanical system. The Poisson bracket of two functions $f(\mathbf{q}, \mathbf{p})$ and $g(\mathbf{q}, \mathbf{p})$ is defined to be

$$\{f, g\} = \sum_{i=1}^{n} \left(\frac{\partial f}{\partial q_i} \frac{\partial g}{\partial p_i} - \frac{\partial f}{\partial p_i} \frac{\partial g}{\partial q_i} \right).$$

Show that

(1) $\{q_i, q_j\} = \{p_i, p_j\} = 0$ and $\{q_i, p_j\} = \delta_{ij}$

(2) A new set of coordinates and momenta $\mathbf{Q} = \{Q_1, Q_2, \ldots, Q_n\}^T$, $\mathbf{P} = \{P_1, P_2, \ldots, P_n\}^T$ defined as

$$\begin{pmatrix} \mathbf{Q} \\ \mathbf{P} \end{pmatrix} = A \begin{pmatrix} \mathbf{q} \\ \mathbf{p} \end{pmatrix}$$

via a constant $2n \times 2n$ matrix A still satisfy the relation in (1), if and only if $A \in Sp(2n, \mathbb{R})$.

Problem 05

Construct a homomorphism from the quaternion group to the Klein four group,

$$\phi : Q \to \mathbb{Z}_2 \times \mathbb{Z}_2.$$

Show its kernel $\ker \phi$ and image $\mathrm{im}\phi$.

Problem 06 (p. 26 of [GM])

Show that the following diagram commutes if and only if $k_1 = k_2 \mod N$.

$$\begin{array}{ccc} \mathbb{Z}_N & \xrightarrow{m_{k_1}} & \mathbb{Z}_N \\ \downarrow \psi & & \downarrow \psi \\ \mu_N & \xrightarrow{p_{k_2}} & \mu_N \end{array}$$

[1] Greg Moore notes, Chapter 01, http://www.physics.rutgers.edu/ gmoore/618Spring2022/GTLect1-AbstractGroupTheory-2022.pdf